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When Supply Meets Demand: The Case of Hourly Spot Electricity Prices

Alexander Boogert
Birkbeck, University of London

Dominique Dupont
University of Twente

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Alexander Boogert* and Dominique Dupont†

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Abstract

We use a supply-demand framework to model the hourly day-ahead spot price of electricity based on publicly available information. With the model we can forecast the level and the probability of a spike in the spot price defined as the spot price being above a certain threshold. Several European countries have recently started publishing day-ahead forecasts of the available supply. In this paper we show potential uses of such indicators and test their forecasting power in an hourly spot price model. We conclude that a forecast of the available supply can be part of a useful indicator and discuss ways to further improve the forecasts.

1 Introduction

Day-ahead spot electricity prices provide an important reference point to all members of the electricity industry. These prices are characterized by high volatility and rare but violent spikes. These aspects have motivated significant research efforts. In this article we model the spot electricity price based on the supply-demand equilibrium.

There are several ways spot electricity models can be applied. In short-term tactic planning it is important to forecast the absolute height of the day-ahead spot prices and to forecast the probability of a spike. On a long-term basis the variability of spot prices becomes interesting as well. This variability can be used as an input for the long-term valuation of powerplants. In this article we focus mainly on the short-term horizon.

Our goal is to establish a relation between several fundamental drivers and hourly spot electricity prices. In particular we will investigate the role of available capacity can play to explain hourly spot electricity prices. Using hourly prices instead of daily prices has two advantages. In the first place it gives an explanation for the different shapes of the prices over the day. In the second place it increases the sample size and hence the likelihood of obtaining robust empirical results. Accurate forecast of demand and supply is of paramount importance to the electricity industry because these two must be balanced at any time to maintain the stability of the power grid. Forward electricity contracts are traded several years before actual delivery. Contracts are traded both on the OTC market and on organized exchanges and delivery is normally channelled through a day-ahead market. The market design differs between electricity markets. Examples of design differences include the exact time of settlement, the granularity of the contracts (i.e, the time period

*Birkbeck College, University of London, Commodities Finance Centre & Essent Energy Trading, 's-Hertogenbosch, The Netherlands.

†Corresponding author. FELab, University of Twente, Capitool, P.O. Box 217, 7500 AE Enschede, The Netherlands.

for which power is to be delivered), the handling of actual delivery in real-time and the exact information provided to the public. In general day-ahead markets take the form of an auction. First, the independent auctioneer aggregates buy and sell orders from the different market participants for electricity to be delivered the following day for each individual hour. Then it computes 24 market-clearing prices for the next day, which are the day-ahead prices (or spot prices) we discuss in this article.

Some of these market design differences, like the exact time of settlement and granularity of the market, do not have a great impact on the relation. Markets settle at different times in the morning creating small differences in available information used for the relation. Most markets operate at an hourly granularity, while the UK and Australia operate at an half-hourly granularity. On the other hand, it is worth noting that the relation between day-ahead and real-time markets differs significantly between electricity markets. Longstaff & Wang (2004) find in the PJM market in the United States that power prices on the day-ahead market are on average higher than on the real-time market and relate this spread to several risk factors. Karakatsani & Bunn (2005) find that in the UK market the difference shows a diurnal pattern. Boogert & Dupont (2005a) find that in the Dutch market differences are rarely positive on average and are always characterized by very large potential losses. Other differences include the type of information available and its quality. For example, is there a day-ahead estimate of available supply, and do these numbers cover the full market? The great variety of market setups makes it necessary to adapt models to the local conditions to make them useful. However, these local adjustments generally do not affect the cores of the models.

The remainder of this article is as follows. In section 2 we review the literature. In section 3 we establish the supply-demand framework for a general electricity market. First we discuss factors which influence spot electricity prices. Then we introduce a relation between supply, demand and spot prices and show how it can be used for forecasting hourly spot prices and the probability of a spike. Subsequently, we contrast our non-parametric approach with some parametric ones. In section 4 we introduce the situation in the Dutch market. We specify which data is available and apply the techniques for forecasting an hourly spot price and the probability of spike. The section ends with a study on the stability of the relation. In section 5 we discuss the implications for further modelling.

2 Review of the literature

The modelling of electricity spot prices has long focused on the reduced form models (e.g. Cartea and Figueroa (2005) and Huisman and Mahieu (2003)). Two popular modelling approaches are jump diffusion and regime switching. Both types of models are mathematically tractable and have received considerable attention. In particular, the estimation of those models tends to be delicate. Another route is provided by fundamental models (e.g. Kosecki (1999)), which carefully describe the characteristics of the supply stack in a market. In the case of a central planner the full supply stack is known and used to serve the load at the lowest cost. In liberalized markets only the general shape of the daily supply stack is known. The marginal cost curves one obtains with fundamental modelling and estimates of the supply stack need to be transformed into spot prices.

A hybrid model incorporates ideas from the two approaches. Compared to reduced-form models, hybrid models take into account useful additional information besides the price time series like for example load, weather or availability of power plants. Eydeland and Geman (1998), Eydeland and Geman (1999), Pirrong and Jermakayan (1999), Pirrong and Jermakayan (2000), Skantze, Gubina and Ilic (2000) and Eydeland and Wolyniec (2003) are examples of a class of hybrid models based on the assumption that there is an exponential relation between price and load. This captures the behavior of strongly increasing prices when the load is growing, while it can facilitate closed form solutions for the pricing of electricity derivatives.

Derivative pricing is based on continuous time models, while forecasting is mainly based on discrete time models. Both types of models can be hybrid. Our model can be seen as a discrete time model. We will discuss the connection to continuous time models in more detail in section 3.6. The approach we develop can be used in both discrete and continuous time models.

The main ingredient for our model is the reserve margin. Reserve margin denotes the fraction of the total supply which is still available for covering the demand (or: available capacity). In practice, there are several definitions possible of the available capacity, which we will discuss in more detail in section 3.1.

Besides spot electricity modelling, the reserve margin is studied in research on public policy (e.g. Visudhiphan & Ilic (2000)) and security of supply (e.g. Birnbaum et al. (2002)). In spot electricity modelling reserve margin is studied in Anderson (2004), Burger et al. (2004), Mount et al. (2006) and Zareipour et al. (2006). Anderson (2004) prescribes a functional form for the relation between a type of reserve margin and the probability of a spike. Burger et al. (2004) prescribe a functional form for the relation between an index related to the reserve margin and the spot price together with residual short-term fluctuations and long-term variation of prices. The index incorporates the expected relative availability of power plants and load, though the precise form is not given. Mount et al. (2006) create a regime switching model where the switching probabilities between the regimes and the conditional means for each regime vary with time and with reserve margin. Zareipour et al. (2006) discuss how different variables and different models can improve the forecast of the day-ahead market prices. They find a variant of the reserve margin is a useful indicator in the Ontario market.

3 The supply-demand framework

In this section we discuss factors which influence spot electricity prices. Besides past spot electricity prices, there is a range of factors which could impact the analysis. As one of our goals is to investigate the forecast of the available capacity, we dedicate the first subsection to this topic. Subsequently, we discuss additional price drivers.

3.1 Forecasting available capacity

In most electricity markets accurate information is available on the amount and the price of power traded on the market in the past. By construction, supply equaled demand at those prices. More information about the state of the market can be obtained from the supply and demand curve. In some electricity markets there is a clear relation between price and volume on the day-ahead market because all supply has to be offered in the day-ahead market (e.g. in the old NETA system in England and Wales or currently in Spain). In these markets one can use the supply and demand curves as input about the state of the market. However, there are also markets where not all supply is offered in the day-ahead market and subsequently there is no apparent relation between price and volume. For example, there is no clear relation at the Dutch market (APX) as can be seen in figure 1. The APX represents only about 20 percent of the total national load.

An alternative to the transacted volume on the day-ahead market is the system load. The system load is the demand for power within some area. The relation between system load and market price is known to be stronger, which is confirmed by the sample we are using as can be seen in the middle panel in Figure 4. The bidding curve at the day-ahead market gives the demand (and supply) curves as a function of price. This makes it easy to determine the demand elasticity, but it presents only the demand within the day-ahead market. The derivation of the demand elasticity from a single number like the system load is more difficult.

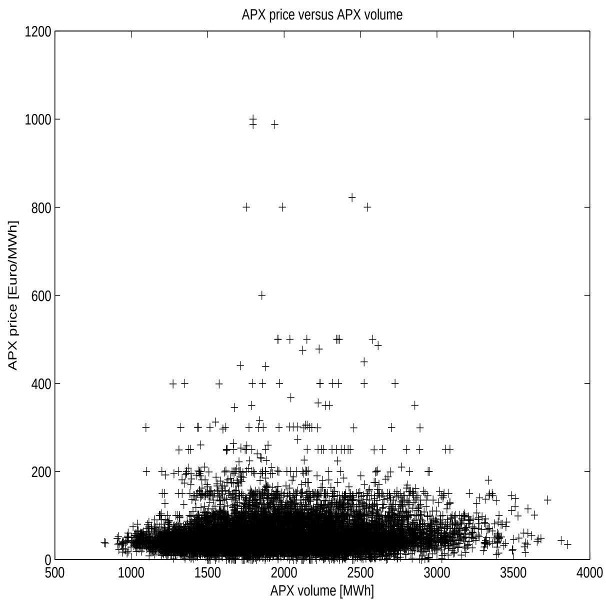

Figure 1: APX price versus APX volume

Another reason for the absence of data comes from a delay in data release. For example, in the PJM market the supply and demand curves are released with a delay of 6 months (see Mount et al. (2006)). On top of that, Mount et al. note that the available capacity could not be fully recovered from the public data on offered capacity and an assumption on the total available capacity was necessary.

In markets where the information from the day-ahead market cannot be used, the elasticity of supply and demand has to be determined by other means. One measure for the supply elasticity is the spare capacity available. This capacity crucially depends on the granularity of the market. In the very short run (e.g. within 15 minutes) only some flexible units can be turned on and the output of running units can be increased. On a day-ahead basis more of the capacity will be available.

Demand elasticity is normally not taken into account as consumers are generally price insensitive. However, there are groups of large consumers (e.g. in the metals industry) for whom it is possible to temporarily decrease their power intake. These customers are willing to reduce their demand in exchange for a reduction of the electricity price. In financial terms: the customers sell an interruptable contract. There is no public data available on this type of contracts. These demand elasticity effects are explicitly taken into account in Fezzi & Bunn (2006). In this article we refrain from this effect, and assume it will enter the model implicitly.

Within the existing literature most articles do not explicitly introduce supply because of an apparent lack of data. Recently, the situation has improved as indicators have been introduced in several European electricity markets. Regulators are currently providing estimates for the available capacity in The Netherlands, UK and Germany. In this article we will focus on the Dutch market, which has the longest history of these three.

Another grey area in the definition of available capacity is the use of import and export capacity. The question is how to include potential import and export into the total available generation capacity. An important difference is created by the timing of the import/export capacity market in comparison to the day-ahead markets, and the timing between day-ahead markets in different markets. In case the capacity auction is in advance of the day-ahead market, the resulting price is an indicator for the day-ahead spot price. To be more precise: it is an indication for the upcoming spread between two different day-ahead spot electricity markets.

3.2 Additional price drivers

Natural price drivers are factors which impact supply or demand or both. Besides there can be feedback effects from prices in either the previous spot price or the most recent real-time prices. To give an indication about the variety of potential price drivers, we refer to Hughes & Parece (2002). They mention power supply factors (installed capacity, outages, generation resource mix, transmission constraints), demand factors (load duration, weather sensitivity, economic activity, retail price) and market design (retail price caps, revenue share of spot sales, capacity requirements and wholesale price caps) as possible price drivers.

3.3 A relation between supply, demand and spot price

One of the goals of this article is to understand the relation between supply, demand and the spot price. We focus on a simple economically motivated relation which allows us to study the underlying data carefully. Alternatively, one could apply data mining techniques like neural networks and adaptive splines (see e.g. Lu et al. (2005)). Inspired by Anderson (2004) we have decided to consider a demand-supply ratio (DSR) of the following kind:

$$\text{DSR} := 1 - \frac{\text{demand}}{\text{available capacity}} \tag{1}$$

This index is a reserve margin index: it covers the fraction of the supply which is available for covering demand. It is realistic to believe this index will have a negative correlation to the spot prices. We note that the lower the index, the less capacity is available and the tighter the market. This will then imply that more expensive units are coming online, and marginal costs increase. In addition, we expect the bandwidth around the average to increase for lower indices. Note that this index is closely related to the concept of capacity utilization as used by Anderson (2004). Capacity utilization states how much of the available capacity is used to cover the demand, that is: capacity utilization equals 1 - reserve margin.

There are two natural candidates which could provide an alternative to our index. Instead of a demand-supply ratio, we could opt for a supply-demand ratio or an absolute difference between the supply and demand:

$$\frac{\text{available capacity}}{\text{demand}} - 1 \tag{2}$$

$$\text{available capacity - demand} \tag{3}$$

As mentioned by Visudhipan & Ilic (2000), the supply-demand ratio is more sensitive to variation in supply than the demand-supply ratio, while the demand-supply ratio is more sensitive to variation in demand than the supply-demand ratio.

We have chosen to use the demand-supply ratio for the following reasons. In the first place, market participants indicated a preference for this ratio. In the second place, we face mainly measurement errors in supply which play a lesser role in the demand-supply ratio than in the supply-demand ratio or the absolute difference between the supply and demand. In the third place, we prefer a ratio as it becomes dimensionless.

We believe this improves stability of the relation. Note that in the end we consider a relation from a dimensionless quantity to Euros.

3.4 Forecasting hourly spot prices

One way to forecast the spot price is to consider the average relation from reserve margin to hourly spot prices. Simultaneously this allows us to produce a confidence interval around the forecast. From an economic viewpoint we expect this relation to increase for a decreasing reserve margin. As well, we expect the bandwidth to increase for decreasing reserve margin. In our empirical part, we have used both a smoothed *b*-spline fit and a piecewise linear fit.

The bandwidth is an interval forecast of the price, which appears to receive attention only since recent. For example, Misiorek et al. (2006, p.23) claim "interval forecasts have not been investigated to date." The natural extension of our relation is to consider a two-dimensional version of this approach. Such a step was taken in Lu et al. (2005) where besides a reserve margin a steepness-of-load indicator was used.

3.5 Forecasting the probability of a spike

Besides the absolute height of the spot prices, an important variable for market participants is the probability that the prices will end up above a certain threshold. We will call a spot price above the threshold a spike. In this article we define the threshold as a fixed amount of euros. An alternative would be to define the threshold in terms of the cost of the marginal unit for the specific hour under consideration. We estimate the probability as the relative number of observations above the threshold in our data sample.

At this moment it is interesting to relate our work to Anderson (2004). She put a parametric relation from reserve margin to spike probability at a central point in the model and kept the relation fixed. This motivated us to study the stability of the relation in the empirical part. In the following paragraph we discuss more parametric models to contrast such an approach to our non-parametric approach.

3.6 Parametric approach

In our approach we assume there is a non-linear relation between the reserve margin and spot prices. Another approach is to parametrize the relation. In a functional form, we can rewrite our reserve margin index as follows:

$$S_t = f\left(1 - \frac{D_t}{C_t}\right) \tag{4}$$

where S_t is the spot price for hour t, D_t is the demand, C_t is the available capacity and f is a non-linear function.

The variable $1 - \frac{D_t}{C_t}$ takes values between 0 and 1, and S_t can take very high values. If one assumes a monotonic relation between reserve margin and price, it is reasonable to base f on the inverse of a cumulative distribution function (cdf) with infinite support and given in closed form, for example, the logistic distribution. This appears (the function is given without explicit motivation) to be the motivation behind Anderson (2004). Similarly, Barlow (2002) makes the power price a function of a latent variable that follows a diffusion process (this variable need not be between 0 and 1). The function is built to contain a singularity, which pushes the price towards infinity in the neighborhood of the singularity. The inverse cdf technique can be seen as refinement of this technique.

Alternatively, one can treat supply and demand as separate stochastic processes, and introduce a functional form for the relation of the form $S_t = f(D_t, C_t)$. Using this functional form and an explicit link between day-ahead and forward prices, creates a possibility to study the forward risk premium. Bessembinder and Lemmon (2002) formulated a general equilibrium model for the day-ahead forward prices, which they applied to the PJM market in the US. Villaplana (2005) extended the model by considering supply as a random variable and applied the model to the Nordpool market in Scandinavia. In these models the relation is assumed to be of exponential or power form. This creates the possibility to estimate the parameters by a linear equation, and allow for closed form solutions for forward prices. Simultaneously, it captures the empirical phenomenon that prices rise for increasing demand and decreasing capacity. To give an example, Villaplana (2005) estimates

$$S_t = \gamma_1 C_t^{\gamma_2} e^{\gamma_3 D_t} \tag{5}$$

where γ_1, γ_2 and γ_3 are constants.

A version related to our approach was independently created by Kanamura and Ohashi (2007). Besides a parametrization by a Box-Cox transformation, they parametrize the relation between price and load by two linear and one quadratic function. This comes close to our piecewise linear fit.

4 Application to the Dutch market

We first discuss the structure in the Dutch market and the availability of data. Second, we describe how this data behaves and show how to make a forecast for the spot price and the probability of a spike. Finally, we discuss the stability of our relation.

4.1 Overview of the Dutch market

The Netherlands was among the first countries in the European Union to liberalize its electricity market. The Dutch ISO, TenneT, manages the high-voltage grid (380 and 220 kV), which interconnects regional electricity networks and links the Dutch grid to Belgium and Germany. TenneT, a wholly state-owned company, ensures access to the domestic high-voltage network and organizes, through its subsidiaries, the day-ahead market for electricity (Amsterdam Power Exchange or APX) and the imbalance market. It also auctions capacity at the five cross-border interconnectors. The maximum import in normal circumstances is 3650 MW, which can be increased to 3850 MW in case of emergencies. The scheduled day-ahead import is not exactly realized in real-time. Although the electricity traded on the APX represents about 20 percent of the Dutch daily consumption, the APX price is considered an important benchmark.

In the Dutch market import/export capacity is auctioned before the day-ahead spot electricity and imported electricity has to be offered on the day-ahead market. It is worth noting that there is a need for import to the Dutch market and that the available import capacity is used frequently.

A new development is the introduction of market coupling between the Netherlands, Belgium and France (Belpex (2006)). Under this new system the import/export auction will be integrated into the day-ahead auction. This will present a new challenge for the derived relation.

4.2 Available data in the Netherlands

Before 2004 no public estimates were made for the available supply in normal circumstances. Only when available capacity dropped below a low threshold was the public informed about the state of the system. As this happened rarely, it was difficult to estimate the available capacity and the demand-supply equilibrium in general. Boogert & Dupont (2005b) show that in that period the water temperature was a good indicator of the spike risk in the electricity price: hot water reduces potential capacity and hot water occurs when temperatures are high leading to high demand.

Since 2004, TenneT publishes an estimate for the available capacity for the coming 30 days in the Dutch grid. TenneT gathers statements of the different generators about the availability of their individual plants, and summarizes them on an aggregate level. The TenneT estimate covers most of the generation in the Netherlands. Two sources of generation are not present: potential wind generation and generation in smaller units (less than 10 MW). The TenneT estimate is one way to describe the supply-demand equilibrium in the Netherlands. On top of that estimate (which we will denote by TAC) we think the following types of data could be related to the supply-demand framework:

NL National load: realized generation including realized net import gives the load which is published by TenneT on 15 minute basis with a delay of two days. There is no official forecast available. Note the official load data covers only electricity generated by units larger than 10 MW.

RIE Realized import or export: history published by TenneT on 15 minute basis. We take import as a positive number since it adds to the available capacity. This information is published with a delay of 30 minutes.

MI Maximum import: the maximum possible import and export is published by TenneT. A day-ahead forecast is available, together with announcements for future maintenance and enlargement in case of emergency. We received a historical database from TenneT.

WP Wind power: there is no official estimate of the total wind production in the Netherlands. An internal estimate was provided by Essent Energy Trading.

In this report we are working with data starting 01/10/2004 and ending 17/06/2006.[1] The starting date coincides with the first publishing date of the forecast for the available capacity. As spot prices are published on an hourly scale, we transformed all 15-minute data into hourly data by taking the average over that specific hour. Subsequent graphs all show hourly data. In total there are 14904 hourly data points.

As mentioned in section 3.1, there are several ways to define available capacity. In the Dutch market, we need to make two choices. The first deals with whether we should include realized imports (or exports) or the day-ahead forecast of maximum imports (or exports). The second choice deals with whether we should include wind power capacity.

The potential for wind energy is growing in the Netherlands. In 2004 the total installed capacity was 1073 MW, which grew to 1224 MW in 2005 (CBS, 2005). Given its size, it could be interesting to include wind power into the total available capacity. However, as the data is not public, we have for the current version decided to exclude wind power from the available capacity. Concerning the import/export number, we have chosen to use the day-ahead forecast of maximum possible import/export. Real-time flows are not available day-ahead and flows appear more a resultant of our model. Thus we work with the following estimate for the total available capacity C_t:

$$C_t = TAC_t + MI_t \tag{6}$$

[1] For convenience we deleted the four days with daylight saving hours in our sample: 31/10/2004, 27/03/2005, 30/10/2005 and 26/03/2006.

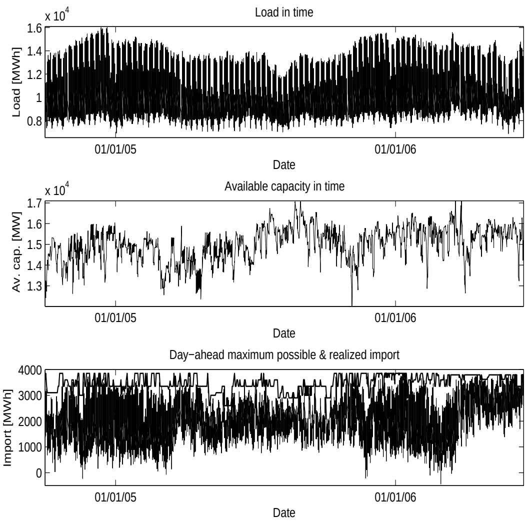

Figure 2: National load, available capacity and day-ahead maximum possible import capacity. In the bottom panel we include realized net import during our data sample

4.3 Reserve margins in the Netherlands

Let us start with showing the development over time of the underlying data for the reserve margin. In Figure 2 we show the load, forecasted available capacity and day-ahead maximum possible import capacity. To contrast we also include the realized values of the import into the bottom panel. In the figure we see load changes between 700 and 1600 MWh, while the forecast of the available capacity moves between 1200 and 1700 MW. The realized import varies significantly, while the maximum possible import capacity varies much less.

In Figure 3 we show the development over time of both the APX price and the reserve margin. A scatter plot in the upper panel of Figure 4 reveals a pattern of increasing prices with reserve margin. For completeness, the figure also graphs the APX against the national load (the middle panel) and against the available capacity (the lower panel). It is good to note that there are some high prices for still medium index values. This will become more apparent in the following paragraphs and we discuss the stability of our relation in section 4.6.

4.4 Forecasting the spot price

Given the reserve margin, one way to forecast the spot price is to consider the average relation from reserve margin to APX prices. In Figure 5 we show a piecewise linear fit and a *b*-spline fit. The piecewise linear fit was created by a discretization of the reserve margin. We create intervals of width 0.05, and take the average of all spot prices within each interval. Reserve margin takes values between 0.10 and 0.70, leading to 12 intervals $(0.10 - 0.15, 0.15 - 0.20$, etc). We denote an interval by its ending point (so the first interval is 0.15). To show the impact of the width of the interval we included a piecewise linear fit with an interval of width 0.005.

From the figure it is clear that odd humps can occur in the piecewise linear fit. Apparently a width of

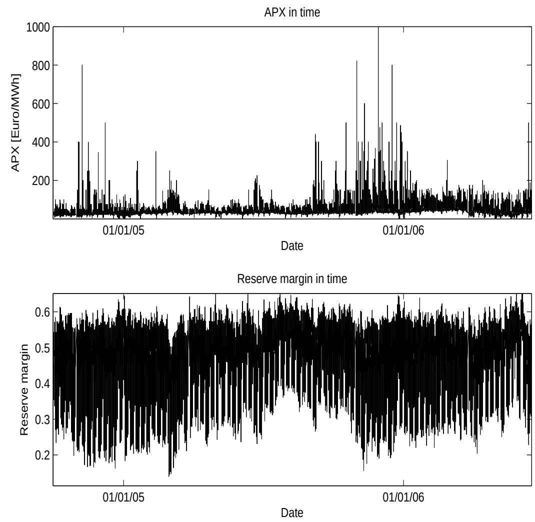

Figure 3: APX price and reserve margin during our data sample

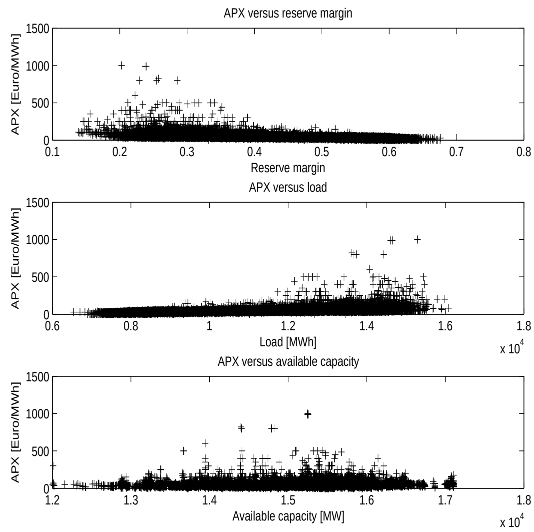

Figure 4: Scatter plots of APX versus respectively national load, available capacity and reserve margin

0.05 led to a monotonic decreasing fit for the existing data set. As a counter example, consider the fit using the width 0.005, which contains several wiggles. These wiggles contradict the economic intuition that prices should increase as reserve margins decrease.

One way of creating a fit which complies with economic theory is to fit the data under the constraint it is smooth and monotonic. As can be seen above another method would be to play with the width until the fit is smooth and monotonic again. Fortunately, there exist non-parametric smoothing techniques where monotonicity can be imposed, e.g. in Ramsay (2003). In Figure 5 we show a b-spline (order 6, 15 evenly spaced knots) fit.[2] We subsequently smoothed the fit (lambda=0.02, fourth derivative), which in our case gave a smooth and monotonic fit.

Two interesting points arise from this figure. In this example the piecewise linear fit and the b-spline fit were very similar. In addition, we see that the smooth *b*-spline fit appears rather linear for the reserve margin where most observations occur: values between 0.20 and 0.60. A similar observation was made by Visudhiphan and Ilic (2000) in the NEPOOL market.

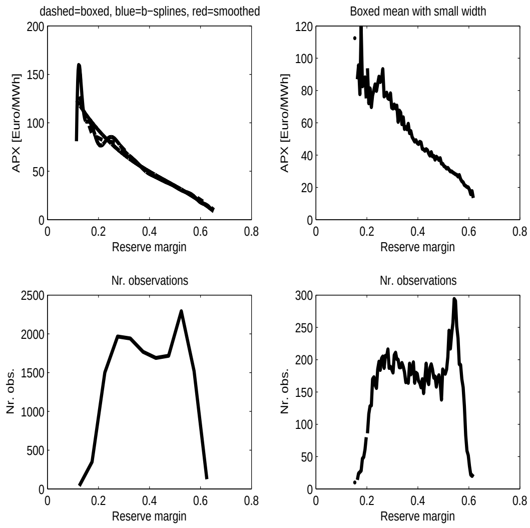

Figure 5: Interpolated graph of the APX-index relation via b-spline, smoothed b-spline and piecewise linear fit. The lower diagram shows the number of underlying observations in each box for normal (left) and small (right) width

Our next question is how spread the real spot prices are around the average relation. In figure 6 we show the variability around the fit by the 5 different percentiles (10, 30, 50, 70, 90) of the relation between the APX and the reserve margin. From the figure we can conclude the price spread is decreasing with the reserve margin. Again we see a hump around index 0.30.

To get a better feeling for the distributions, we give the summary statistics for each of the intervals in table 1. In the table we see standard deviation, skewness and kurtosis are rising for decreasing reserve margin if we disregard the first and the last interval. This is reasonable considering the limited number of data points in these intervals.

[2]The *b*-spline regression is performed with software provided by Jim Ramsay on his website: ftp://ego.psych.mcgill.ca/pub/ramsay/ which includes b-spline, smooth b-spline and smooth, monotonic b-spline

interval	nr obs	mean	std	skew	kurt
0.15	39	127.16	63.49	1.79	5.76
0.20	348	87.80	87.03	5.20	43.08
0.25	1501	81.15	73.06	5.84	56.84
0.30	1966	79.64	50.10	4.17	39.24
0.35	1940	65.42	36.67	4.25	37.94
0.40	1767	52.63	22.83	2.75	19.20
0.45	1689	43.97	16.44	2.28	13.85
0.50	1717	38.02	12.37	2.09	17.73
0.55	2291	30.39	9.98	0.42	5.50
0.60	1519	24.90	9.65	-0.18	3.10
0.65	125	16.10	9.57	-0.22	1.75
0.70	2	13.70	19.35	0.00	1.00

Table 1: Summary statistics for different intervals: number of observations, mean, standard deviation, skewness and kurtosis

4.5 Forecasting the probability of a spike

In this article we define a spike as a price above 90 euros, which is in line with market practice. We will take the threshold as given, and do not include it as one of the parameters to be estimated. A threshold of 90 euros implies that about 11% of the data sample is qualified as a spike. In table 2 we give the percentage of the complete data sample that would be qualified as a spike for some other threshold choices.

APX threshold	80	90	100	120	150	200
Exceeding probability	0.1410	0.1084	0.0760	0.0485	0.0214	0.0086

Table 2: Different exceeding probabilities for different threshold levels. In the full sample there are 14904 points.

In figure 7 we show the relation between the probability of a spike and the reserve margin. We take as the probability of a spike, the relative number of observations in a specific reserve margin interval above 90 euros. By varying the threshold we found similar graphs as the ones presented. Again, we see our data does not follow the economic theory: the probability is not strictly increasing for a decreasing reserve margin.

For comparison we included the spike probability function proposed by Anderson (2004). Although estimated in another market, we see that our data spikes earlier and that the cut-off point is not as clear as in the PJM data. As shown by Birnbaum et al. (2002) and Mount et al. (2006) for the PJM market and Ilic and Visudhiphan (2000) for the NEPOOL market, the probability of spike rises fast for reserve margins below 20%.

4.6 Stability

The non-monotonicity of the relation between reserve hourly margin and wholesale electricity price leads us to investigate the stability of the relation. In particular, we consider the time dependence on the daily and

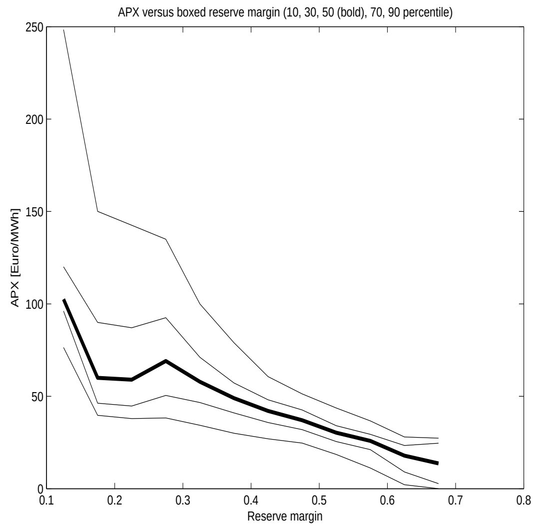

Figure 6: The top diagram shows the average relation occurring for the different intervals together with the 10, 30, 70 and 90 percentile. Note the 50 percentile is the median.

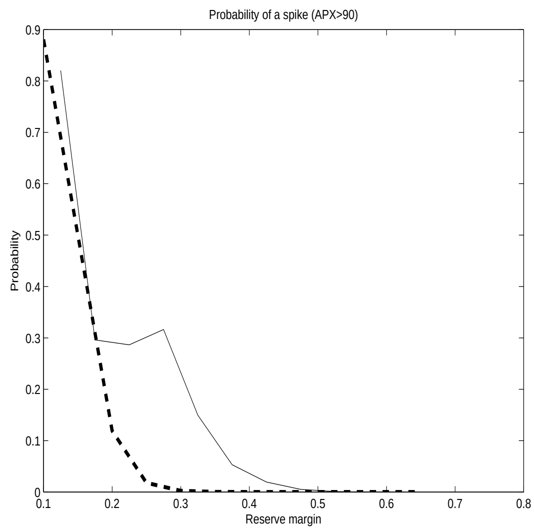

Figure 7: Probability of a spike versus reserve margin. The dashed graph is proposed by Anderson (2004) for the PJM market

the yearly level, one-off events and out-of-sample performance. Before we start, let us give some reasons the relation may be unstable.

4.6.1 Reasons for instability

The relation between reserve margin and spot prices combines information about all hours irrespective whether it is day or night, week or weekend. As we approach individual hours, the relation between adjacent hours is not explicitly taken into account. This information is especially relevant in situations where load is quickly changing or reaches its peak, as start-up costs start to play a role too at those moments. Because the start-up costs for the power plant to cover the maximum load need to be earned back, prices become temporarily higher than the levels we would normally establish.

Start-up cost also play a role in the weekends. As always, an operator must choose between incurring start-up costs and incurring a loss on the electricity (sell below marginal costs). Shutting down during the weekend is normally not the most economic choice, even though the load can vary a lot. This implies the demand factor moves and the prices can drop below normal levels for certain moments.

A similar must-run situation can occur in the winter with power plants that produce both heat and power. In order to cover the heat demand, the power plants become must-run in electricity. With the effect of must-run units known, one can make the hypothesis that hours which are covered with must-run units have a lower price.[3] To test this hypothesis, we should be able to make the distinction between flexible and inflexible (must-run) units in the forecast of available supply. Until now, this type of split is not available in the Dutch market. Therefore we will test the implication of our hypothesis that, for the same level of reserve margins, wholesale power prices are lower in the weekends.

4.6.2 Dependence on time of day

In this and the next subsection we check whether the relation is similar among different subsets of the data. Here we consider the usual time-of-day subsets. Note that the definition of such subsets (like peak and off-peak) differ across markets. Table 3 describes the definitions in the Dutch market.

Products	Hours	Nr. Observations
Baseload	0-23	14904
Off-peak	0-6+23	4968
Peak	7-22	9936
Weekend-peak	7-22 (weekend only)	2800
Shoulder	7+20-22 (week only)	1784
Super-peak	8-19 (week only)	5352

Table 3: Description of different time of day segments and the number of observations. Baseload denotes the full data set.

In Figure 8 we see that peak and off-peak prices are in line, though off-peak prices fall below peak prices for a reserve margin between 0.40 and 0.50. This is in line with our hypothesis. In Figure 9 we consider the average relation for the three subsets of the peak: weekend-peak, shoulder and super-peak. In the figure we see that keeping reserve margin constant, the power price tends to be lower during shoulder hours than

[3] In the Netherlands negative prices can only occur in the real time market. The minimum possible price at the day-ahead price is 0.01.

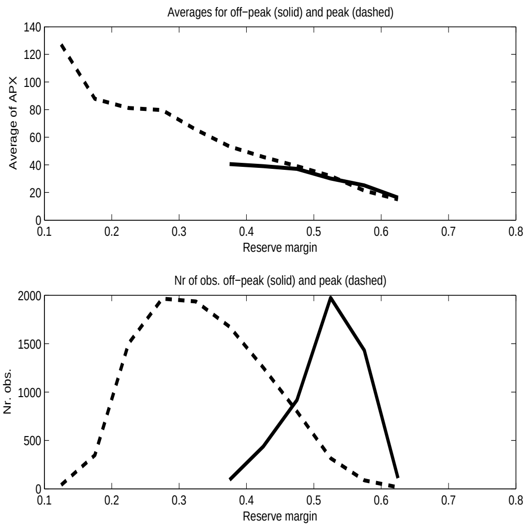

Figure 8: The average relation occurring for peak and off-peak together with the number of underlying observations in each box. To show significant values, we suppress a time of day in a certain box if it contains less than 10 observations of that kind

super-peak or weekend-peak hours. In addition, the weekend-peak prices rise above super-peak prices for a reserve margin below 0.30. This appears out of line with our hypothesis. Since the current picture does not fully explain the behavior of power prices, we investigate seasonal effects in the next section.

4.6.3 Dependence on season

We divided the data into Summer (April-September) and Winter months (October-March). In our data set of 14904 data points, we have 8640 data points in the Winter and 6264 in the Summer. In figure 10 we see that the difference between peak and off-peak is sustained if we split the data in Summer and Winter. The same conclusion holds for the relation between weekend-peak, shoulder and super-peak hours as can be seen in figure 11. This brings us to the conclusion it is better to specify a separate model for week and weekend days.

4.6.4 Outliers

In the previous section, we have seen that certain data points were not in line with the average relation. This leads to the question whether something strange, e.g. a learning period, has happened in our data sample. For this reason we consider how these "outlier" data are distributed over the data sample.

We decided to look at data points which have high APX values, and relatively high values of reserve margin. For example, if we use the (arbitrary) definition of an outlier as $APX > 200$ & reserve margin > 0.3, we call 16 data points outliers while in total 128 data points had $APX > 200$. In Figure 12 we graph how many of such data points were clustered in one day over time. We see there are two days with three outliers (that is: on two days there were three hours which are rather out of line from the usual behavior), and that outliers are mainly present around October 2005. For comparison reason, we included again the

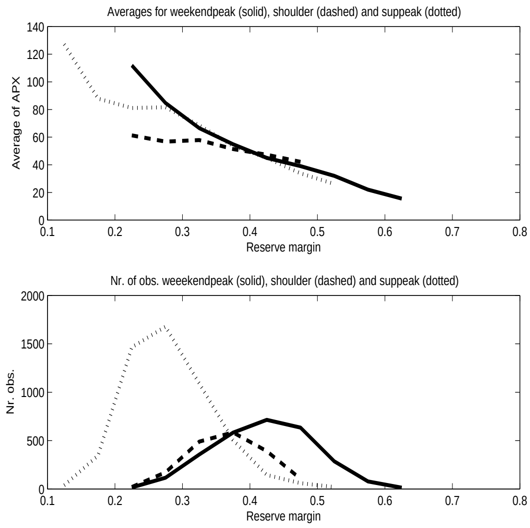

Figure 9: The average relation occurring for three different time of day indices together with the number of underlying observations in each box. To show significant values, we suppress a time of day in a certain box if it contains less than 10 observations of that kind

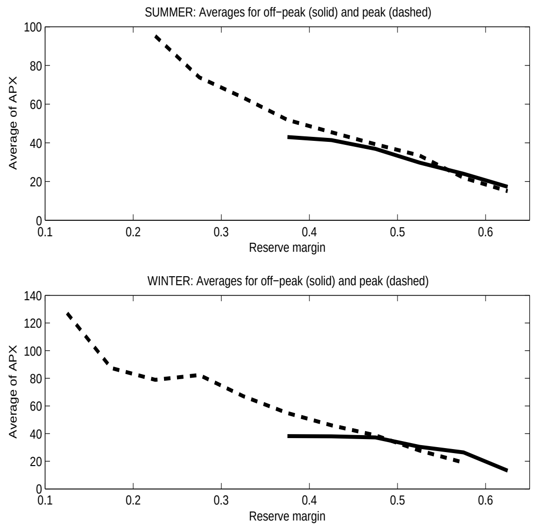

Figure 10: The average relation occurring for peak and off-peak indices for both summer and winter. To show significant values, we suppress a time of day in a certain box if it contains less than 10 observations of that kind

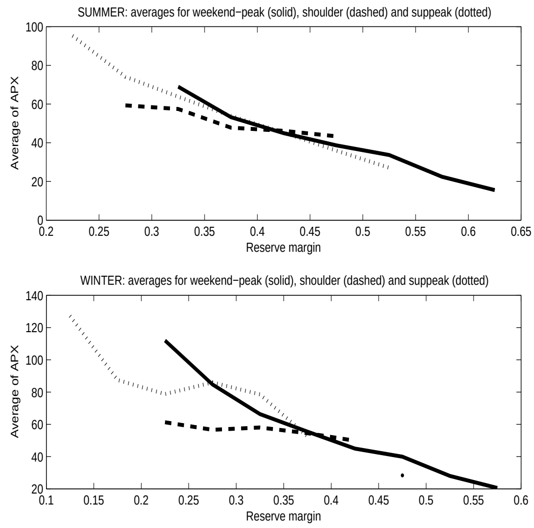

Figure 11: The average relation occurring for three different time of day indices for both summer and winter. To show significant values, we suppress a time of day in a certain box if it contains less than 10 observations of that kind

development of APX prices over time. In specific, we can conclude the hump was not due to odd data in the beginning of the data sample, which indicates no learning period.

4.6.5 Out-of-sample

Up to now, we have used the whole sample to draw conclusions about the relation between the reserve margin and the spot prices. In this subsection, we give a first indication how stable the relation is over different parts of the sample. In other words: are there different pricing regimes over time? An alternative to our approach could be a regime discovery algorithm proposed by Vucetic et al. (2001).

For our stability check we divide our sample in three parts (the first 5000, the second 5000 and the remaining 4904 observations). In figure 13 we compare the average price and the probability of spike. We see that the average price has increased, and that the relation from the first period understates the average price and probability of spike in the second period. The relation from the second period is close to the relation of the third period. This would show a good out-of-sample behavior. Part of the increase has been due to an increase in marginal costs. This has not been captured by our current definition of a spike.

One way to incorporate the increase of prices would be to include an average error over a certain period. Then, the natural question is how many data points we should use in our data estimation by comparing the errors out-of-sample. We will not address this question in this paper.

5 Discussion and conclusion

In this article we have shown how to create an estimate for the supply-demand framework and how to build a simple model around it. One of the main findings is that reserve margin matters and should be included

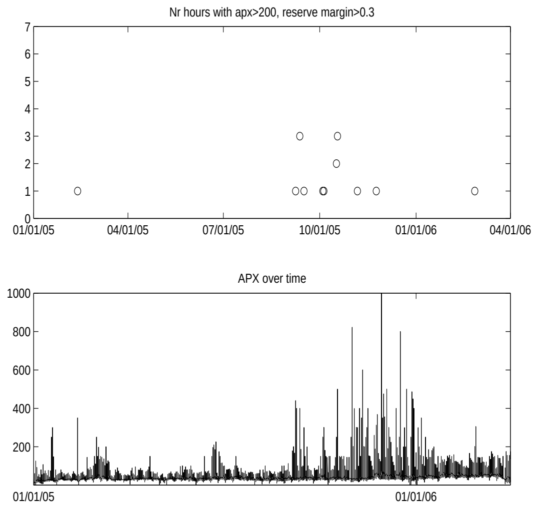

Figure 12: The top panel shows how many hours on a specific day had both a high APX price (> 200) and high reserve margin (> 0.2). The bottom panel shows APX prices.

into a spot electricity model to enhance performance. In practice, another useful area of application has shown to be the development of a fundamental model. While most fundamental models can create estimates of future marginal costs, it needs a link from marginal costs to market prices. Our model can provide such a link if marginal costs are driven by reserve margin.

Estimates for available capacity are not public data in all the different electricity markets. Due to the simple nature of our approach, we believe it will be easy to replicate our results in other markets. In Europe, two examples would be the UK and German market which both started to publish estimates. The UK market is most similar to the Dutch market. For the German market, it is important to look carefully at the inter-connectors and wind production.

Our piecewise linear fit can contain a double hump structure which does not comply with economic theory. As well, our results imply the Dutch market can spike for a still medium values of the reserve margin. This could be a proof of unreliable estimates for the available capacity or natural noise in the market. A step forward could be made if we could group available capacity by technology, but this information was not available.

The backbone of our relation is an assumption on stability. We have studied the stability over different time of days and seasons. We found it is better to specify a separate model for different time of days, where especially it is worth to split week and weekend days. Out-of-sample test gave promising initial results. To improve the relation, we think to include an autoregressive part for the error in the spot price prediction for the previous period. This is left for future research.

The model can be extended in different directions. One of the directions is into the relation between spot and forward. With a stability assumption it is possible to simulate different underlying drivers and create a simulation of future spot prices. This road has been followed by Anderson (2004). The study of forward risk premia in a similar perspective has been performed by specifying a functional form for the relation between supply, demand and spot prices. Another direction is the extension to a coupled market. This type

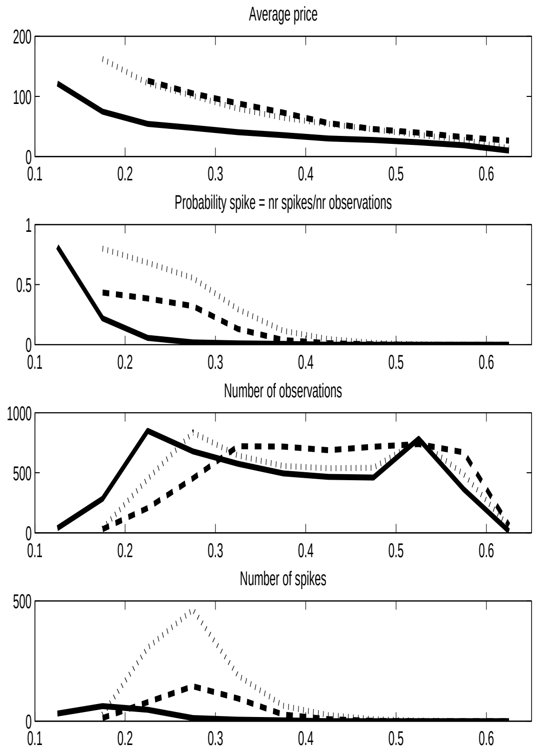

Figure 13: Average price for three different time periods (period 1: solid, period 2: dashed, period 3: dotted) together with the probability of spike, the number of observations and the number of spikes

of markets are present in the US and in Nordpool. As indicated by Belpex (2006), the Dutch, Belgian and French market will be integrated too. This will provide a new challenge for the current model.

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